

Theory of the Resistive Transition in Overdoped $Tl_2Ba_2CuO_{6+\delta}$: Implications for the angular dependence of the quasiparticle scattering rate in High- T_c superconductors.

V. B. Geshkenbein^{1,4}, L. B. Ioffe^{2,4} and A. J. Millis³

¹ *Theoretische Physik, ETH Hönggerberg, Zürich, Switzerland*

² *Department of Physics and Astronomy Rutgers University, Piscataway, NJ 08854*

³ *Department of Physics and Astronomy, The Johns Hopkins University
3400 North Charles St., Baltimore, MD 21218*

⁴ *Landau Institute for Theoretical Physics, Moscow, Russia*

We show that recent measurements of the magnetic field dependence of the magnetization, specific heat and resistivity of overdoped $T_c \sim 17K$ $Tl_2Ba_2CuO_{6+\delta}$ in the vicinity of the superconducting H_{c2} imply that the vortex viscosity is anomalously small and that the material studied is inhomogeneous with small, a few hundred Å, regions in which the local T_c is much higher than the bulk T_c . The anomalously small vortex viscosity can be derived from a microscopic model in which the quasiparticle lifetime varies dramatically around the Fermi surface, being small everywhere *except* along the zone diagonal (‘‘cold spot’’). We propose experimental tests of our results.

The nature of the superconducting transition in high- T_c superconductors is the subject of present controversy: some workers argue that the transition is driven by thermal or quantal fluctuations of the superconducting order parameter [1,2] implying that in a wide regime above the resistively determined T_c one has local superconducting order without global phase coherence. In such a situation one would expect strong superconducting fluctuations, i.e. a large paraconductivity, magnetoresistance and fluctuation diamagnetism. The apparent absence (or weakness) of these effects in all but the most heavily underdoped compounds casts doubt on the phase fluctuation hypothesis [3].

Recent measurements on overdoped $T_{c0} = 17K$ $Tl_2Ba_2CuO_{6+\delta}$ in a magnetic field provide an interesting new perspective on this issue. There is a reasonably sharp resistive transition with onset temperature $T_\rho(H)$ [4–6]. A pairing onset temperature $T_\gamma(H)$ can be defined for $H < 4$ T from specific heat and magnetization measurements [7,8]. At $H = 0$, $T_\rho = T_\gamma = T_{c0}$, the zero field transition temperature, but at $H > 0$ it is found that $T_\rho(H) < T_\gamma(H)$, implying that the resistive transition is due to vortex lattice melting. However, in the range $T_\rho(H) < T < T_\gamma(H)$ the reported resistivity has a very weak temperature and field dependence, i.e. it is possible in this material to destroy superconductivity by phase fluctuations without producing a strong paraconductivity or magnetoresistance. The goal of this paper is to understand in more detail this interesting phenomenon.

The $Tl_2Ba_2CuO_{6+\delta}$ materials have also attracted attention because the resistively determined upper critical field $H_\rho(T)$ has an anomalous temperature dependence [4], curving sharply upwards as T is decreased below $5K$. It has been widely assumed that the upward curvature is an intrinsic property of the material [9], however we will argue that the low- T behavior is due to the presence

in the sample of small regions with T_c much higher than the bulk T_{c0} . A closely related idea involving small regions with anomalously large H_{c2} was put forward in the general context of dirty superconductors in [10].

We first discuss the resistivity, specific heat and magnetization data. At low applied magnetic field B the material is superconducting. The resistivity $\rho = 0$ and there is a large ‘London’ magnetization given in equilibrium by $M_{London} = -\Phi_0 \ln(H_{c2}/B)/32\pi^2\lambda^2$. (Real materials are not in equilibrium because the vortex lattice is pinned). At some field $H_{melt}(T)$ (visible e.g. as the foot of the resistive transition in Fig 2 of ref [4]) the vortex lattice (glass) melts. The magnetization is observed to take the London value [8] and the resistivity becomes non-zero. For $H_{melt}(T) < H < H_\rho(T)$ the resistivity has a strong H and T dependence; we interpret this as evidence that the transport is dominated by vortex pinning near the melting transition. Above $H_\rho(T)$ the resistivity saturates and loses its strong field dependence. For concreteness we define $H_\rho(T)$ as the field at which ρ reaches 90% of its saturated value.

The specific heat exhibits a maximum which broadens and shifts to lower T as the field is increased. We define $H_\gamma(T)$ from the temperature of the maximum; it turns out that $H_\gamma(T) \approx 2H_\rho(T)$ in the field range $0 < H < 2$ T in which the maximum is visible. We interpret $H_\gamma(T)$ as the scale at which bulk superconducting pairing vanishes, i.e. as the ‘microscopic’ H_{c2} .

A paired state should exhibit a large magnetization vanishing at the ‘microscopic’ H_{c2} . The observed [8] magnetization is apparently given by the sum of a ‘London’ term with a $\ln(1/B)$ field dependence and a diamagnetic term with a linear B dependence, i.e. $M(B, T) = M_{London} + \chi_d B$. The London term is observed for a range of temperatures greater than $T_\rho(H)$, however as T approaches $T_\gamma(H)$ the second, diamagnetic term dominates

the measured magnetization so the behavior of the London term is difficult to determine. The data are consistent with the expectation that the London term vanishes at $H_\gamma(T)$. The coefficient of the diamagnetic term, χ_d has magnitude much larger than the usual Landau diamagnetism and persists over a wide range of $T > T_\gamma(H)$, indeed up to $T \sim 2T_{co}$. We interpret the diamagnetic term as arising from the presence in the sample of small regions with transition temperatures much higher than the bulk superconducting T_{co} .

The scales $H_{melt}(T)$, $H_\rho(T)$ and $H_\gamma(T)$ are shown in Fig 1, along with a shaded area indicating the region in the $H-T$ plane where an anomalously large diamagnetic term χ_d is observed. Note that a straight-line extrapolation of $H_\gamma(T)$ to $T = 0$ yields a $H_\gamma(0) \ll H_\rho(0)$. The relatively small value of $H_\gamma(T = 0)$ implies that the low-T upturn of H_ρ is not a bulk property of the material. We will show below that it is due to the same inhomogeneities which produce the diamagnetism.

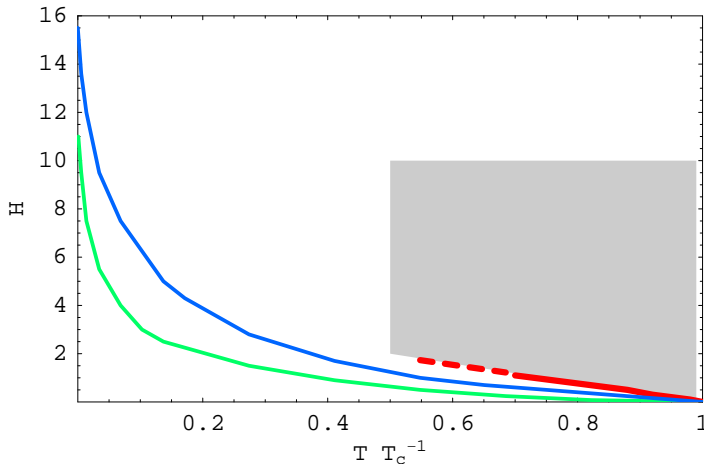


Fig 1 Phase diagram of $T_c \approx 15$ K $Tl_2Ba_2CuO_{6+\delta}$ as determined from resistivity, specific heat and magnetization measurements. Lowest line: H_{melt} ; intermediate line H_ρ , upper line H_γ . Linear diamagnetism was observed in the shaded region and (although not shown in the figure) persists to $T \approx 2T_c$.

From the phase diagram one sees a wide separation between the local pairing scale H_γ and the resistive scales H_ρ and H_{melt} . This implies that the whole temperature range between $T_{melt}(H)$ and $T_\gamma(H)$ should be described as a vortex liquid, albeit one with unconventional transport properties. The conventional view [11] of transport in a type II superconductor in a non-zero magnetic field is as follows. One has vortices, these move in response to an applied current and this motion causes the phase of the superconducting order parameter to become time dependent leading via the Josephson relation to an electric field which causes dissipation and hence a finite conductivity, σ_V . The total conductivity, σ , is the sum of the vortex part, σ_V , and a normal one, σ_n due to uncondensed carriers. The standard estimate of the vortex conductivity

in the flux-flow regime is the Bardeen-Stephen formula

$$\sigma_{BS} = \frac{H_{c2}}{H} \sigma_n. \quad (1)$$

Although it was originally obtained from phenomenological arguments, subsequent work [12,13] based on the quasiclassical kinetic equation for superconductors shows that it is remarkably robust, and applies in almost all situations except very close to the microscopic H_{c2} or in the "superclean" limit.

The interpretation of the Bardeen-Stephen formula is that the density of vortices is H/Φ_0 (Φ_0 is flux quantum) and the dissipation per vortex is the core area (which is Φ_0/H_{c2}) divided by σ_n . At $H \ll H_{c2}$ the number of vortices is small and the vortex conductivity is large, $\sigma_{BS} \gg \sigma_n$. The observed conductivity is then controlled by the vortices which short circuit any conductivity from the normal carriers. It has a $1/H$ field dependence which essentially counts the number of vortices and a temperature dependence which must include a dramatic drop from σ_n to σ_{BS} as the temperature is reduced below $T_\gamma(H)$. Neither the $1/H$ field dependence nor the dramatic resistivity drop below T_γ is observed in $Tl_2Ba_2CuO_{6+x}$. Further, the number of vortices cannot differ significantly from the mean-field estimate because the Ginzburg parameter is small, of order 10^{-2} [14].

Therefore, we conclude that the dissipation per vortex in $Tl_2Ba_2CuO_{6+\delta}$ is much less than that predicted by the Bardeen-Stephen formula. We note that an anomalously small vortex viscosity has been directly observed in terahertz experiments on YBCO films [15].

The dissipation due to vortex motion has been considered by many authors. A result for 2D d-wave superconductors with circular Fermi surface and an angle independent quasiparticle lifetime has been derived by Kopnin and Volovik [13]. Generalizing their result to include an angular dependent lifetime, $\tau(\theta)$, and density of state ν yields

$$\sigma_V = \frac{1}{n_V} \langle \gamma(\theta) \rangle_\theta \quad (2)$$

Here $\langle \rangle_\theta$ denotes an average over the 2D Fermi surface parametrized by angle θ and

$$\gamma(\theta) = 2\nu(\theta)\Delta(\theta)^2\tau(\theta) \ln \left(\frac{\Delta_{max}^2}{\Delta^2(\theta)} \right) \quad (3)$$

Here Δ is the superconducting gap. If $\tau(\theta) \equiv \tau$ has negligible angular dependence then $\langle \gamma(\theta) \rangle_\theta$ may be re-expressed as $H_{c2}\tau$ and the equation for σ_V becomes the usual Bardeen-Stephen one. Deviations from the Bardeen-Stephen form occur when τ has a strong angular dependence. From Eq 3 we see that the dissipation due to moving vortices is determined mainly by the lifetime of the particles in the regions where the gap is large. This will differ from the dissipation due to the normal state conductivity, σ_n , if the latter is dominated by the zone

diagonals where the superconducting gap vanishes. In the usual Boltzman approach $\sigma_n \approx \frac{e^2}{2\pi} p_F \langle v_F(\theta) \tau(\theta) \rangle_\theta$; this will be dominated by the diagonals if

$$\tau(\theta) = \left(\frac{1}{\tau_0^{-1} + \Gamma \theta^2} \right) \quad (4)$$

with θ the angle measured from the zone diagonal and $\Gamma \tau_0 \gg 1$. This formula may be derived by assuming that the scattering rate is the sum of two terms, a conventional one which is due to the impurities or Fermi liquid scattering which we parametrize by τ_0^{-1} and unconventional one, $\Gamma \theta^2$, which is large everywhere except for the zone diagonals. Evidence for a large Γ comes from photoemission experiments [16] which at $T > T_c$ find dispersing quasiparticle peaks for momenta near the zone diagonals (“cold spots”) but find only very broad incoherent structures for other momenta. From the width of these structures we estimate $\Gamma \gtrsim 0.1 \text{ eV}$. Normal carrier conductivity implied by Eq 4 is $\sigma_n = \frac{e^2}{\pi} v_F p_F \sqrt{\tau_0/\Gamma}$. It is dominated by quasiparticles near the zone diagonals; because the superconducting gap vanishes there σ_n does not change significantly as T is reduced below T_c . In the $Tl_2Ba_2CuO_{6+x}$ materials of interest here σ_n is of order $(10 \mu\Omega cm)^{-1}$ implying that $\sqrt{\Gamma/\tau_0} \approx 4 \text{ meV}$. The photoemission estimate $\Gamma \gtrsim 0.1 \text{ eV}$ along with $v_F \approx 1 \text{ eV/\AA}$ and $p_F \approx 0.5 \text{ \AA}^{-1}$ implies $1/\tau_0 < 0.2 \text{ meV}$. In a separate paper [17] we show that Eq. (4) with $\tau_0 \propto T^2$ and $\Gamma = \text{const}$ reproduces the linear resistivity and T^{-2} Hall angle observed in optimally doped materials.

Assuming $\Delta = \Delta_0 \sin \theta$, $\nu(\theta) = \nu_0$ constant and $\tau(\theta)$ given by Eq (4) gives

$$\frac{\sigma_V}{\sigma_n} = \frac{\int d\theta \sin^2 2\theta \tau(\theta) \frac{H_{c2}}{B}}{\int d\theta \tau(\theta) \frac{H_{c2}}{B}} = \frac{\Lambda}{\sqrt{\Gamma \tau_0}} \frac{H_{c2}}{B} \quad (5)$$

with Λ a number of the order of unity determined by the detailed behavior far from diagonals. The data for $Tl_2Ba_2CuO_{6+x}$ imply that σ_V/σ_n is very small for $B > 0.2 H_{c2}$ requiring that $\sqrt{\Gamma \tau_0} \gg 5$. Our estimates yield $\sqrt{\Gamma \tau_0} \gtrsim 20$.

Our picture may be tested in two ways. The existence of a vortex liquid in the range $H_\rho < B < H_\gamma$ may be established by using heavy ion irradiation to create columnar defects, pinning the vortices and raising the melting line above $H_\rho(T)$. The formula for σ_V/σ_n may be tested by electron irradiation which creates random defects which scatter electrons thereby increasing $1/\tau_0$ and the normal state resistivity, thus increasing σ_V/σ_n and hence the amplitude of magnetoresistance, etc. Furthermore, because $\sigma_n \propto \tau_0^{1/2}$ our model predicts that once the induced disorder becomes greater than the intrinsic one the normal state resistivity should grow as a square root of the irradiation time.

Now we consider the effects of inhomogeneity. We assume that the sample contains grains of transition temperature $T_G > T_{c0}$, size R and spacing d , implying an

areal density $x_G = R^2/d^2$. The magnetization M_G of a grain in a field B is [11]

$$M_G = \frac{BR^2}{48\pi\lambda_G^2} \quad (6)$$

with λ_G the grain penetration depth. The total magnetization is then $x_G M_G$. The experimental result is that at $T = 8K$ and $B = 10 \text{ T}$ (much greater than the microscopic H_{c2}) the diamagnetic magnetization $x_G M_G$ equals the London magnetization observed at the same T and $B = 0.1 \text{ T}$. From this, Eq. (6), the London formula [11] and $H_{c2}(T = 8K) \approx 2 \text{ T}$ we conclude that

$$x_G R^2 = 3 \times 10^4 \frac{\lambda_G^2}{\lambda^2} \text{\AA}^2 \quad (7)$$

We estimate $\lambda_G/\lambda \sim 1/3$ because $\lambda_G(T = 0)$ should be a little less than $\lambda(T = 0)$ and the measurement temperature $T = 8K \sim T_{c0}/2 \ll T_G$; thus $R^2/d \approx 100 \text{\AA}$.

The above analysis assumes that the grain size is larger than the grain coherence length ξ_G which will be less than the bulk coherence length $\xi \sim 100 \text{\AA}$ and assumes that there are no vortices in the grains. Now in order for a vortex to enter a grain the field must be at least large enough that one flux quantum fits inside the grain; then there are additional numerical factors coming from core energy considerations and boundary conditions. Experimentally, the diamagnetism is linear up to at least $10T$; thus $R^2 < \beta \frac{\Phi_0}{10T} \sim \beta 20,000 \text{\AA}^2$ with $\beta > 1$.

The Josephson coupling between two superconducting grains in a normal metal host depends on many details including the temperature, the magnetic field, the size of the grain, the intergrain distance and the strength of the electronic contact between the grain and the host metal [11]. In the dilute limit $d \gg R$ and in zero magnetic field the Josephson energy is

$$E_J(T, \phi) = E_J^0 e^{-d/\xi_n} F_d(\phi) \quad (8)$$

Here $\xi_n = v_F/(2\pi T)$ is the clean limit normal metal phase coherence length. The phase dependence is contained in the function $F_d(\phi)$ which tends to $\sin \phi$ for $d/\xi_n \gg 1$ but becomes a sawtooth function at $d/\xi_n \ll 1$. The Josephson energy scale E_{J0} is

$$E_{J0} = \Lambda^2 \frac{v_F}{2\pi d} (p_F R) N_c \quad (9)$$

Here N_c is the number of planes over which the grains extend and Λ is a number which depends on the geometry and dimensions ($\Lambda \sim (R/d)^{(D-1)/2}$) and on the strength of the electrical contact between the grain and the normal metal. One expects Λ to be rather less than unity.

It is convenient to define $T_0 = v_F/(2\pi d)$; for $d = 2000 \text{\AA}$ and $R = 400 \text{\AA}$, and using $p_F = 0.5 \text{\AA}^{-1}$ and $v_F = 1 \text{ eV/\AA}$ one has $T_0 \approx 1K$ and $E_{J0} \approx 200\Lambda^2 K$. If the host metal remained non-superconducting down to

lowest temperatures, the grains would order at a temperature T_{cg} satisfying $T_{cg} = ZE_J(T_{cg})$ (here Z the effective number of neighbors of a given grain, is ~ 6 at $T > T_0$ and $\sim (T_0/T)^2$ for $T < T_0$). Grains in a normal host provide an explicit realization of a superconducting transition in which T_c corresponds to a loss of phase coherence with local pairing remaining but the temperature dependence of ρ_S is due to quasiparticles and the fluctuation regime is narrow. In the situation of interest here, the superconductivity of the host metal is suppressed by a magnetic field. This affects the intergrain ordering in two ways: it frustrates the phase ordering, leading to a “gauge glass” behavior, and it substantially weakens the coupling between individual grains, by causing interference between different electron paths. The gauge glass effects on the transition temperature may be estimated by replacing Z by \sqrt{Z} . The effect of the magnetic field on the intergrain coupling is much larger and may be estimated by noting that the coupling is dominated by electron trajectories in a tube of width R and length d ; a magnetic field B leads to a flux Φ_B through this tube of order $\Phi_B = BdR$; when this flux is large compared to the flux quantum, the usual interference arguments imply that the coupling is reduced by the factor $\Phi_0/(\pi\Phi_B)$. For $d = 2000 \text{ \AA}$ and $R = 400 \text{ \AA}$, $\Phi_0/(\pi dR) \approx 0.1 \text{ T}$, so the suppression of the coupling in the interesting fields of order 10 T is substantial (factor of 100). To summarize, the field at which the grains order is

$$B_G(T) = \sqrt{Z} \frac{\Phi_0}{\pi dR} \frac{E_J^0}{T} e^{-T/T_0} \quad (10)$$

Our numerical estimates imply that at $T = T_0 \sim 1K$, $B_G \sim 20\Lambda^2 N_c \text{ T}$. In view of the large uncertainties we regard this estimate as reasonable. The temperature dependence (e^{-T/T_0} for $T > T_0$) is certainly in qualitative agreement with the experiment.

To summarize, $Tl_2Ba_2CuO_{6+\delta}$ material exhibits two experimental anomalies: pronounced upward curvature of H_{c2} at low temperatures and a vortex liquid regime with a negligible temperature and field dependence of the conductivity. We argued that the former is not intrinsic but is due to the presence in the sample of small grains with T_c higher than the bulk. We estimated the density and the size of the grains. The latter anomaly is intrinsic and important; it implies that the vortex viscosity is unusually small. We showed that such a small viscosity can arise if the quasiparticle relaxation rate has a very strong anisotropy around the Fermi surface, and in particular becomes very weak along the zone diagonal where the superconducting gap vanishes. We call the places where the scattering rate vanishes “cold spots”. Finally, we note that small viscosity implies large quantum fluctuations of vortices and, perhaps, quantum liquid of vortices at $T = 0$ in a wide field range.

A crucial issue in high- T_c superconductivity is the apparent coexistence in underdoped materials of local pair-

ing over a wide range above the resistive T_c and a mean field like superconducting transition. Our work shows that there are two ways in which this may occur: the vortex liquid contribution to the conductivity may be very small or the local pairs can be confined to ‘grains’ that are coupled only via Josephson junctions through a normal host. It will be interesting to see if these possibilities are indeed realized in *underdoped* materials.

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